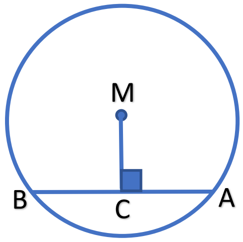
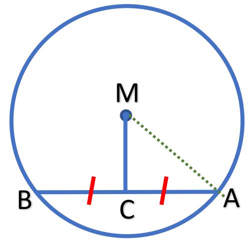
All About the circle



$\because MC \perp AB$

$\therefore AC = BC$

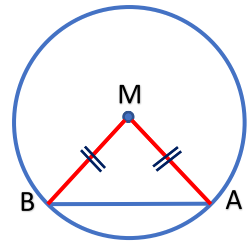
$\therefore C$ *is the midpoint of* $\overline{AB}$



$\because C$ *is midpoint of* $\overline{AB}$

$\therefore MC \perp AB$

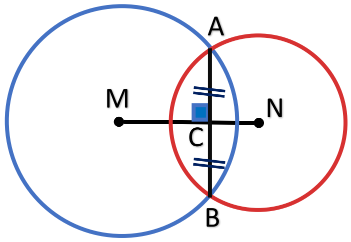
$\therefore \Delta\, MCA$ *is right – angled*



$\because MA = MB = radii$

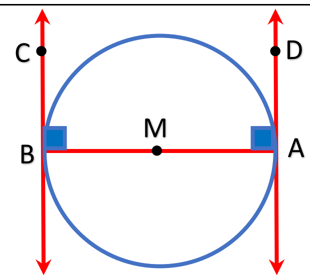
$\therefore \Delta\, AMB$ *isosceles*

$\therefore m(\angle A) = m(\angle B)$



$\because \overline{AB}$ *is a common chord* , $\overline{MN}$ *is line of centers*

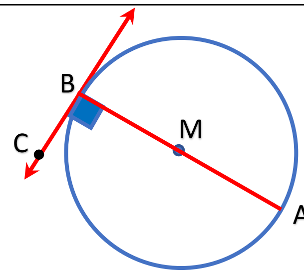
$\therefore \overline{MN} \perp \overline{AB}$, C *is the midpoint of* $\overline{AB}$



$\because \overleftrightarrow{AD}$, $\overleftrightarrow{BC}$ *are two tangents* ,

$\overline{AB}$ *is diameter*

$\therefore \overleftrightarrow{AD} // \overleftrightarrow{BC}$, $\overline{MA} \perp \overline{AD}$, $\overline{MB} \perp \overline{BC}$



$\because \overrightarrow{BC}$ *is a tangent* , $\overline{AB}$ *is diameter*

$\therefore \overline{AB} \perp \overline{BC}$

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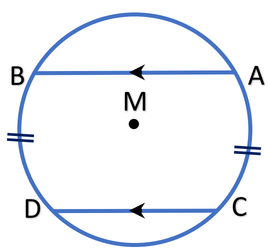
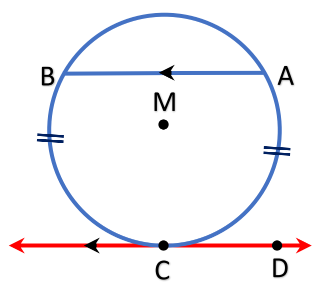
and If $m(\angle B) = 90^o$ $\therefore$ $\overrightarrow{BC}$ *is a tangent*



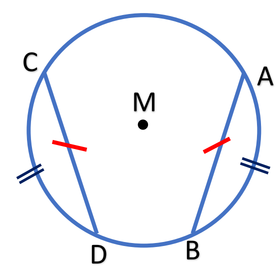
$\because \overline{AB} \,//\, \overline{CD}$

$\therefore\ m(\widehat{AC}) = m(\widehat{BD})$



$\because \overline{AB} \,//\, \overleftrightarrow{CD}$

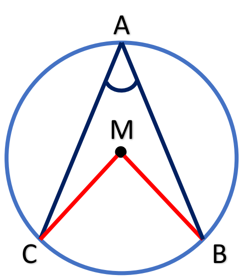
$\therefore\ m(\widehat{AC}) = m(\widehat{BC})$



$\because\ m(\widehat{AB}) = m(\widehat{CD})$

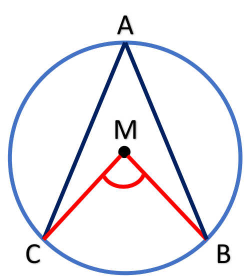
$\therefore \overline{AB} = \overline{CD}$ (*two chords*)

And vice versa



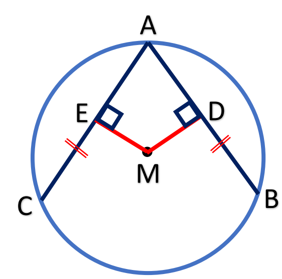
$m(\angle A)$ *inscribed* $= \frac{1}{2}\, m(\angle M)$ *centeral*

$m(\angle A) = \frac{1}{2}\, m(\widehat{BC})$



$m(\widehat{BC}) = m(\angle M)$ *centeral*

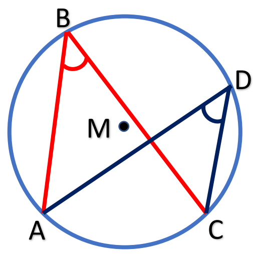
$m(\widehat{BC}) = 2\, m(\angle A)$ *inscribed*



If $AB = AC$

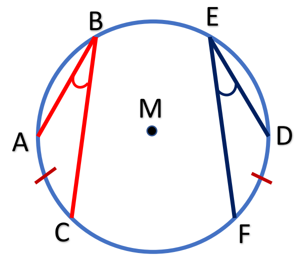
then $MD = ME$

and vice versa



$m(\angle B) = m(\angle D)$

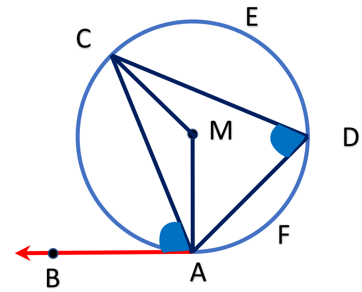
Two inscribed angles subtended by the same arc $\widehat{AC}$



If $m(\widehat{AC}) = m(\widehat{FD})$

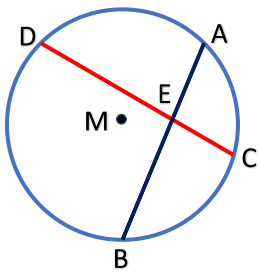
then $m(\angle B) = m(\angle E)$

and vice versa

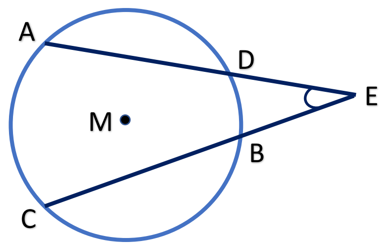


$m(\angle BAC)$ *tangency* $= m(\angle D)$ *inscribed*

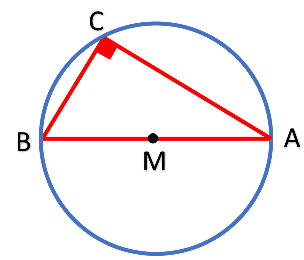
$m(\angle BAC)$ *tangency* $= \frac{1}{2}\, m(\angle M)$ *centeral*



$m(\angle BED) = \frac{1}{2} \times [\, m(\widehat{BD}) + m(\widehat{AC})]$



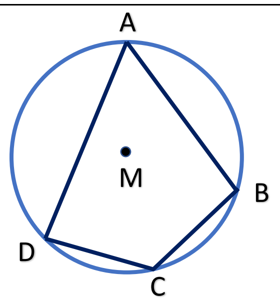
$m(\angle E) = \frac{1}{2} \times [\, m(\widehat{AC}) - m(\widehat{BD})]$



If $\overline{AB}$ *is a diameter*

then $m(\angle C) = 90^o$

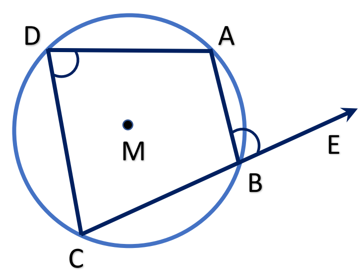
inscribed angle drawn in semi-circle



If ABCD is a cyclic quad.

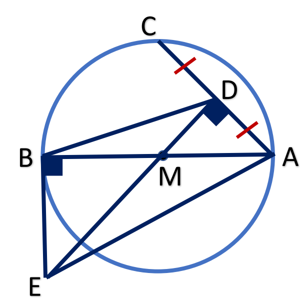
Then $m(\angle A) + m(\angle C) = 180^o$

$m(\angle B) + m(\angle D) = 180^o$



$\because$ *ABCD is a cyclic quad.*

then $m(\angle D) = m(\angle ABE)$ *exterior*



$\because \overline{BE}$ *is a tangent*

$\therefore m(\angle MBE) = 90^o$

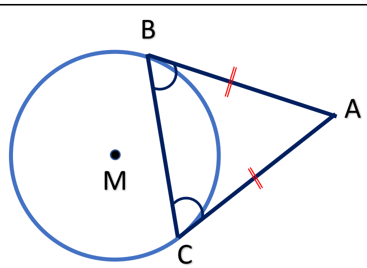
$\because$ *D is the midpoint of* $\overline{AC}$

$\therefore m(\angle MDA) = 90^o$

$\therefore m(\angle ADE) = m(\angle ABE)$

and they drawn on $\overline{AE}$

$\therefore$ *ADBE is cyclic quad.*



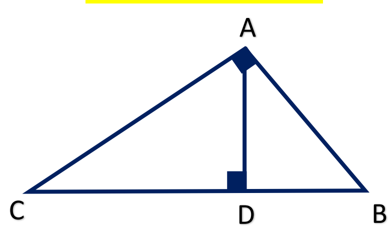
$\because AB = AC$ *(Two tangents)*

$\therefore AB = AC$

$\therefore \Delta\, ABC$ *is an isoscles*

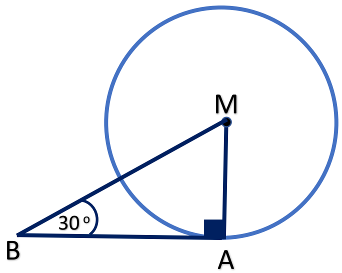
and $m(\angle B) = m(\angle C)$

Euclidean theorem



$\because m(\angle A) = 90^o$ *and* $\overline{AD} \perp \overline{BC}$

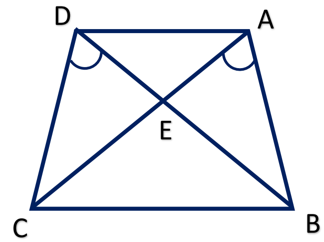
$$AD = \frac{AB \times AC}{BC}$$



$\because m(\angle A) = 90^o , m(\angle B) = 30^o$

$then\ AM = \frac{1}{2} MB$

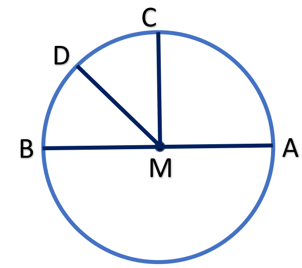
$Side\ opposite\ to\ \angle 30^o = \frac{1}{2}\ hyp.$



$if\ m(\angle BAC) = m(\angle BDC)$

$\therefore ABCD\ is\ a\ cyclic\ quad.$

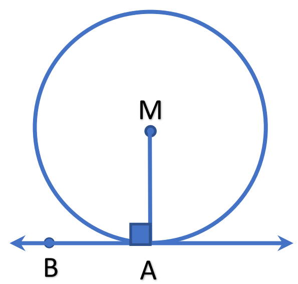
and vice versa



$\because \overline{AB}\ is\ a\ diameter$

$\therefore m(\widehat{ACB}) = 180^o$

$\therefore m(\widehat{AC}) + m(\widehat{CD}) + m(\widehat{BD}) = 180^o$



$\because \overleftrightarrow{AB}\ is\ a\ tangent , \overline{MA}\ is\ a\ radius$

$\therefore \overline{MA} \perp \overleftrightarrow{AB}$

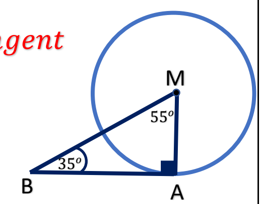
$\therefore m(\angle MAB) = 90^o$

To prove that $\overline{AB}$ is a tangent

Prove that $m(\angle MAB) = 90^o$

Example :

Prove : $\overline{AB}$ is tangent

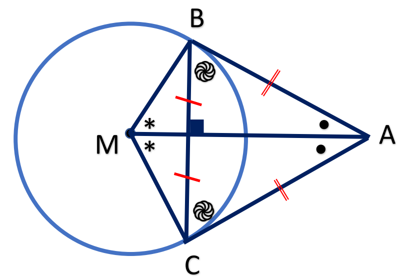


in Δ MAB

$m(\angle MAB) = 180 - (55 + 35) = 90^o$

$\therefore \overline{MA} \perp \overline{AB}$

$\therefore \overline{AB}\ is\ a\ tangent$



$\because \overline{AB} , \overline{AC}\ are\ two\ tangents$

$\therefore AB = AC$

$\therefore m(\angle ABC) = m(\angle ACB)$

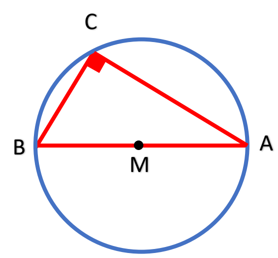
$\therefore \overline{AM}\ bisects\ \angle A\ and\ \angle M$

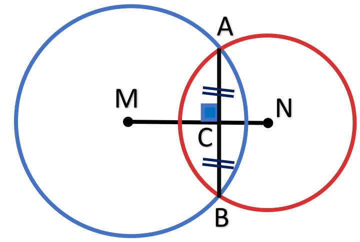
$\therefore AM \perp BC$

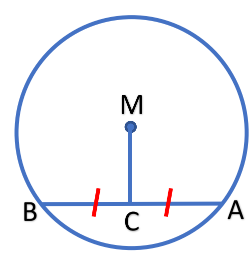
$\therefore ABCD\ is\ a\ cyclic\ quad.$

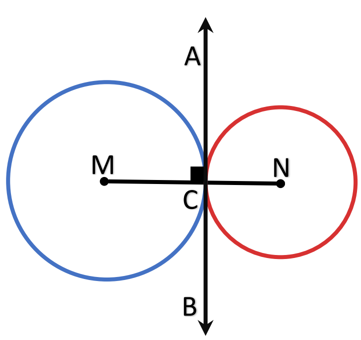
$\therefore m(\angle MBA) = m(\angle MCA) = 90^o$

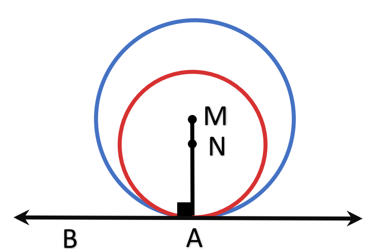
All Right angles

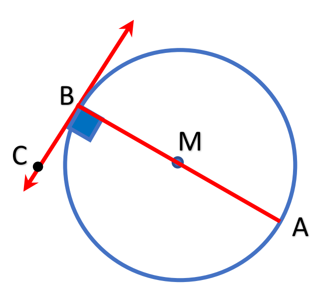






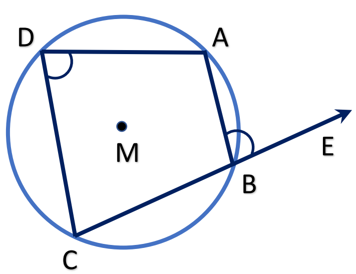






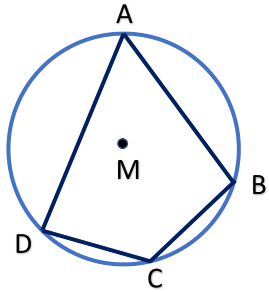
The cyclic quadrilateral

If you know the figure is a cyclic quad. then , you can find one from the following :



The exterioir angle of cyclic quad. equal the interioir angle opposite to its adjacent.

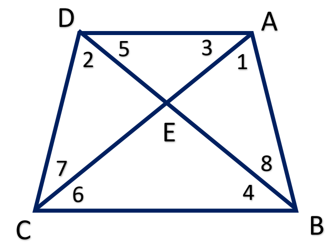
$m(\angle D) = m(\angle ABE)$



Each two oppsite angles are supplementary

$m(\angle A) + m(\angle C) = 180^o$

$m(\angle B) + m(\angle D) = 180^o$



If ABCD is a cyclic quad.

$\therefore m(\angle 1) = m(\angle 2)$ (*two angles drawn on* $\overline{BC}$)

$\therefore m(\angle 3) = m(\angle 4)$ (*two angles drawn on* $\overline{CD}$)

$\therefore m(\angle 5) = m(\angle 6)$ (*two angles drawn on* $\overline{AB}$)

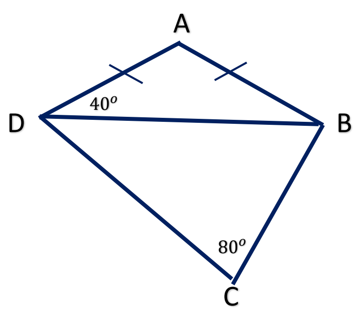
$\therefore m(\angle 7) = m(\angle 8)$ (*two angles drawn on* $\overline{AD}$)

To prove that figure is a cyclic quadrilateral

Find one from the following :

1) ***Prove that two opposite angles their sum =*** 180^o
2) ***Prove that the exterior angle equal to the opposite to the interior angle.***
3) ***Prove that two angles are equal in measure drawn on the same site.***

Ex (1)



Prove: ABCD is a cyclic quad

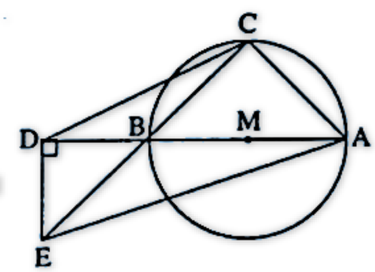
Ex (2)

$\overline{AB}$ is a diameter in the circle M

, $D \in \overrightarrow{AB}$, $D \notin \overline{AB}$, $\overrightarrow{DE} \perp \overrightarrow{AB}$

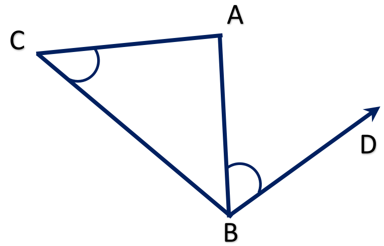
, $C \in \widehat{AB}$, $\overrightarrow{CB} \cap \overrightarrow{DE} = \{E\}$

Prove that : ACDE is a cyclic quadrilateral



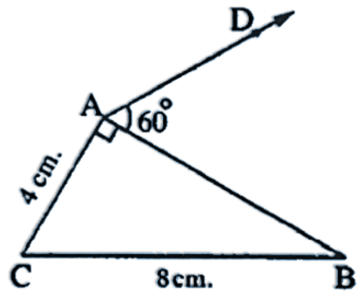
To Prove that $\overrightarrow{BD}$ is a tangent to the circle which passes through $\Delta\ ABC$

prove that: $m(\angle ABD) = m(\angle ACB)$



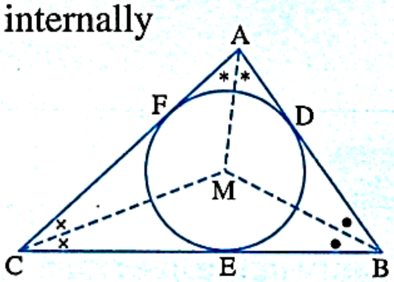
Ex : **Using the given data , prove that :**

$\overrightarrow{AD}$ is a tangent to the circle passing through the vertices of the triangle ABC



The inscribed circle of the triangle

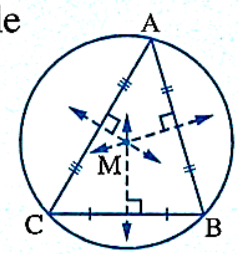
Is the circle that touches all sides of the triangle internally



and its centre is the intersection point of the bisectors of its interior angles

The circumcircle of the triangle

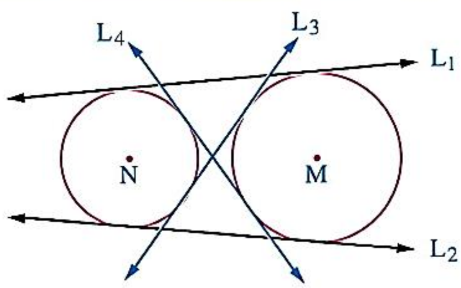
Is the circle that passes through the vertices of the triangle



and its centre is the point of intersection of the perpendicular bisectors of its sides

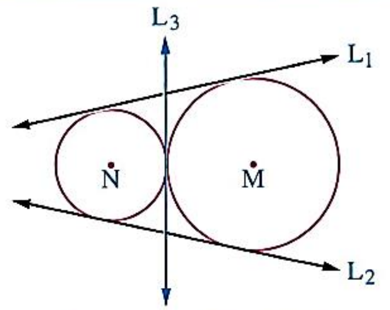
The common tangents to two circles

Two distant circles



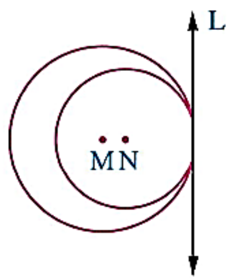
4 common tangents

Two circles touching externally



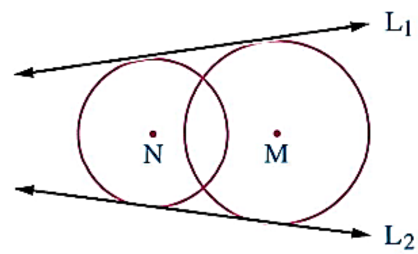
3 common tangents

Two circles touching internally



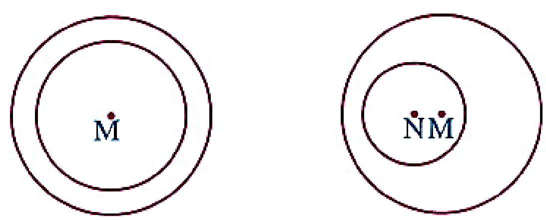
One common tangent

Two intersecting circles



2 common tangents

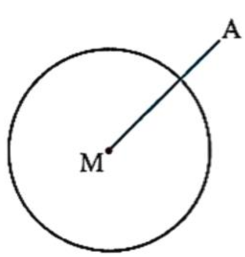
One circle inside the other



There are no common tangents

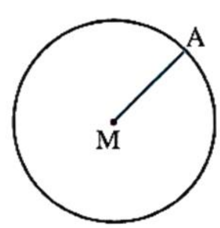
Position of a point with respect to a circle

1 A is outside the circle M



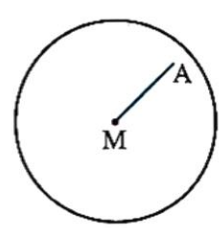
If MA > r

2 A is on the circle M



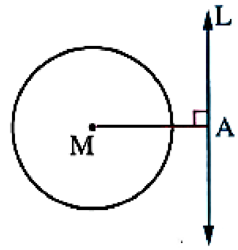
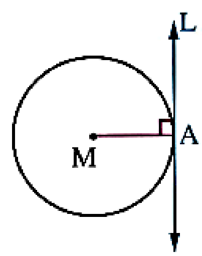
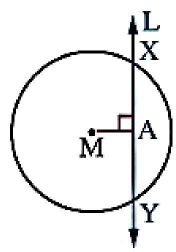
If MA = r

3 A is inside the circle M

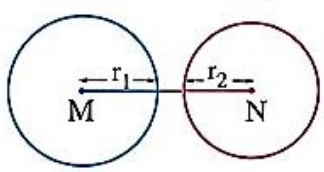
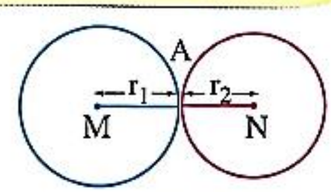


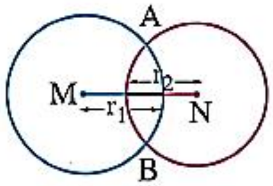
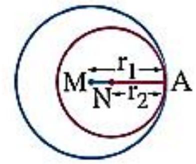
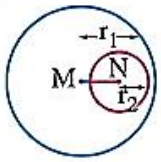
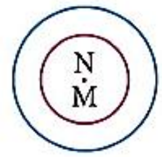
If MA < r

Position of a straight line with respect to a circle

If	Then	The figure	Note that
1 **MA > r**	The straight line L lies outside the circle M		• $L \cap$ the circle $M = \varnothing$ • $L \cap$ the surface of the circle $M = \varnothing$
2 **MA = r**	The straight line L is a tangent to the circle M at A A is called "the point of tangency"		• $L \cap$ the circle $M = \{A\}$ • $L \cap$ the surface of the circle $M = \{A\}$
3 **MA < r**	The straight line L is a secant to the circle M		• $L \cap$ the circle $M = \{X , Y\}$ • $L \cap$ the surface of the circle $M = \overline{XY}$ • $\overline{XY}$ is called the chord of intersection

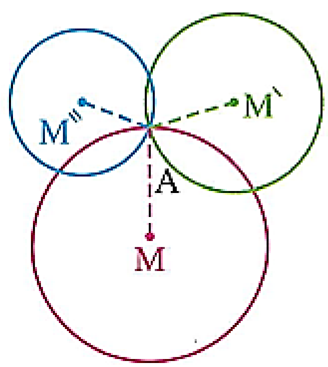
Position of a circle with respect to another circle

If $MN > r_1 + r_2$	If $MN = r_1 + r_2$
	
Then the two circles are : Distant	Then the two circles are : Touching externally

If $r_1 - r_2 < MN < r_1 + r_2$  **Then the two circles are :** Intersecting	**If $MN = r_1 - r_2$**  **Then the two circles are :** Touching internally
If $MN < r_1 - r_2$  **Then the two circles are :** One inside the other	**If MN = zero**  **Then the two circles are :** Concentric

Drawing a circle passing through a given point

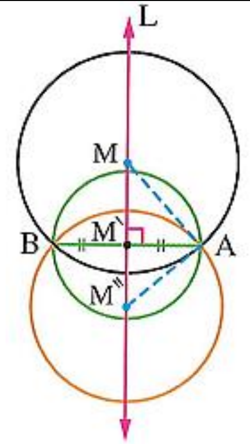
You can draw an infinite number of circles passing through a given point.



Drawing a circle passing through two given points

There is an infinite number of circles that can be drawn to pass through the two points A and B and all their centres lie on the axis of symmetry of $\overline{AB}$

1. If $r > \frac{1}{2} AB$, then we can draw two circles
2. If $r = \frac{1}{2} AB$, then we can draw one and only one circle
3. If $r < \frac{1}{2} AB$, then it is impossible to draw any circle.



Drawing a circle passing through three given points

It is impossible to draw a circle passing through three collinear points.

For any three non-collinear points , there is a unique circle can be drawn to pass through them.

Important rules

Circumference of the circle	$2\pi r$ or $d\pi$
Area of the circle	πr^2
Area of triangle	$0.5 \times base \times height$
Area of parallelogram	$base \times height$
Area of rectangle	$length \times width$
Area of square	$(side)^2$ or $0.5 \times (diagonal)^2$
Area of rhombus	*Base (side)* $\times$ *height* *Or* $0.5 \times d_1 \times d_2$
Area of trapezium	$Middle\ base \times height$ $\left(\frac{base_1 + base_2}{2}\right) \times height$
Perimeter of square	*Side* $\times$ *4*
Perimeter of rhombus	*Side* $\times$ *4*
diagonal of square	$\sqrt{2 \times area}$
Length of the arc	$\frac{measure\ of\ arc}{360} \times 2\pi r$